Noname manuscript No.
(will be inserted by the editor)

Transmission Expansion with Smart Switching Under Demand Uncertainty and Line Failures

Kathryn M. Schumacher · Richard Li-Yang Chen · Amy E.M. Cohn

Received: date / Accepted: date

Abstract One of the major challenges in deciding where to build new transmission lines is that there is uncertainty regarding future loads, renewal generation output and equipment failures. We propose a robust optimization model whose transmission expansion solutions ensure that demand can be met over a wide range of conditions. Specifically, we require feasible operation for all loads and renewable generation levels within given ranges, and for all single transmission line failures. Furthermore, we consider transmission switching as an allowable recovery action. This relatively inexpensive method of redirecting power flows improves resiliency, but introduces computational challenges. We present a novel algorithm to solve this model. Computational results are discussed.

Keywords transmission expansion planning · robust optimization · transmission switching

1 Introduction

Environmental concerns have motivated many governments to require that an increased amount of power be supplied by renewable sources. In the United States, most states have enacted renewable portfolio standards legislation

K. Schumacher
Operations Research, General Motors Research and Development, Warren, MI, USA.
E-mail: kathryn.schumacher@gm.com

A. Cohn
Industrial and Operations Engineering Dept., University of Michigan, Ann Arbor, MI, USA.
E-mail: amycohn@med.umich.edu

R. Chen
Quantitative Modeling and Analysis Dept., Sandia National Laboratories, Livermore, CA, USA.
E-mail: rlchen@sandia.gov

which mandate the fraction of energy generation which must come from renewable sources [4]. Renewable generation has many environmental benefits, but these non-dispatchable sources of power pose a challenge for planners due to the uncertainty in their power output. Additionally, new trends in the areas of demand response, plug-in hybrid electric vehicles and distributed generation are changing the profile of electricity demand. There is uncertainty in the future demand levels, especially when planning over a long-time horizon. Methods for dealing with this uncertainty must be used when planning where to build new transmission lines.

Furthermore, as a society that is increasingly reliant on digital technologies, an uninterrupted power supply is as important as ever. Designing a system that is resilient to failures is critical. However, building new transmission lines to provide redundancy is very expensive. It is important that transmission expansion decisions be made intelligently so as to minimize the total investment costs while also ensuring that the system is robust to failure events. In response to a failure event, i.e., a *contingency*, it is important that a set of feasible actions be available to the operator that allow demand to be met, to prevent a blackout event.

Traditionally, the recovery actions available to the operator include the ability to change generator dispatch levels and influence transmission line power flows. To realize the goal of operating a system that is both resilient and efficient, *transmission switching* has been proposed as a an additional recovery action. Transmission switching refers to the practice of opening a circuit breaker on select transmission lines, effectively temporarily removing the transmission line from the network.

There is a paradox associated with the existence of any transmission line; the line provides a path on which power can be transmitted, but it also imposes a constraint on how power can be transmitted along other transmission lines. In situations where that constraint is acting as a bottleneck, transmission switching can help direct the flow of power to where it is needed. Transmission switching enables transmission lines to be treated as dynamic resources rather than static resources, thereby increasing system flexibility.

As some critics of transmission switching have noted, existing circuit breakers are intended to be used rarely, primarily to de-energize a line that must be repaired. The practice of using circuit breakers as a controllable element may require additional investment in equipment that is designed to be used repeatedly and is remotely controllable. The problem we consider here is thus how to make decisions about where to build new transmission lines *and* where to build new transmission switching equipment. We seek to solve a robust version of the problem where the total investment cost is minimized, and feasible operation is guaranteed for all contingencies and demands within the defined uncertainty sets, given that transmission switching may be used as a recovery action.

The structure of this paper is as follows. In Section 2 we review the relevant literature. In Section 3 we formally define the deterministic transmission expansion problem and develop the robust counterpart. In Section 4 we describe how a cutting plane algorithm could be used to solve the robust transmission expansion problem, describe the development of an *oracle* which returns an unsurvivable contingency-demand pair given an investment solution, and describe how the oracle can be utilized in the cutting plane algorithm. Computational results are presented in Section 5. Finally, Section 6 contains concluding remarks and future work.

2 Literature Review

Transmission expansion planning has been a rich area of research for several decades. In most early works, only dispatchable conventional generation is considered (i.e., uncertain renewable energy is not included) and demand forecasts are assumed to be accurate. Latorre et al. [22] and Romero el al. [27] review several types of deterministic models for the transmission expansion planning problem.

More recently, there has been interest in incorporating uncertainty into the transmission expansion optimization models. A review of the transmission expansion area in general, including a presentation of a few models which incorporate uncertainty, is provided in [30].

Several studies have used stochastic methods to deal with uncertainty in renewable generation and/or the demand for power. Hemmati et al. [16] and Yu et al. [36] consider a transmission expansion planning problem where there is uncertainty in both the demand and the power generated at wind farms. In both works, the authors assume that demand is normally distributed and that wind speeds are distributed according to a Weibull distribution, and they use a Monte Carlo simulation to approximate the probability distribution for the power output at the wind turbine generators. Yu et al. [36] present a chance constrained formulation in which the model seeks a minimum cost expansion plan where the probability of meeting demand is at least equal to a specified threshold. They suggest a genetic algorithm which can return a heuristic solution to the chance constrained formulation. Hemmati et al. [16] propose a multi-objective model to solve for a transmission expansion solution that simultaneously minimizes investment cost, maximizes social welfare, and minimizes loss-of-load. They suggest a particle swarm algorithm to solve the proposed model. One downside of the approaches proposed in [16] and [36] is that the Monte Carlo simulations required to generate the wind power probability distribution are computationally intensive.

López et al. [24] present a model that solves for both transmission and generation expansion decisions when there is uncertainty in demand. This model seeks to minimize the expected cost of both investment costs and operational costs. A set of possible demand scenarios and their probabilities are assumed to be given.

In contrast to stochastic optimization methods that require knowledge of probability distributions, which are generally difficult to ascertain, robust optimization has been used to solve for transmission expansion solutions that are feasible for a variety of demand and/or renewable generation conditions.

Wu et al. [33] propose a robust model of transmission expansion where only uncertainty in demand is considered. The authors use a box uncertainty set for the demand (i.e., an 'interval model'). They propose a branch-and-bound procedure to solve for the worst case demand for a given expansion plan, and use this routine within a Greedy Randomized Adaptive Search Procedure (GRASP) to solve for a heuristic transmission expansion solution.

Yu at al. [35] apply the Taguchi's Orthogonal Array Testing (TOAT) method to the transmission expansion planning problem where there is uncertainty in both renewable energy output and demand. The authors use a box uncertainty set for both demand and renewable generation. TOAT is used to identify a subset of scenarios which are representative of the set of all possible scenarios, as defined by the extreme points of the box uncertainty set. The authors demonstrate that by using only these representative scenarios within a genetic algorithm, they can identify an expansion solution that is robust for most values in the uncertainty set, though it is not guaranteed to be feasible for all demand and renewable generation in the uncertainty set.

Jabr [17] propose a traditional robust model for transmission expansion where there in uncertainty in both renewable generation and loads using two different types of uncertainty sets: a box uncertainty set and a budget uncertainty set. The author proposes a Benders' decomposition procedure. The method that we propose here is similar in spirit to that proposed in [17], although we additionally include uncertainty in line failures, as well as transmission switching as a recovery action.

Outside of traditional stochastic programming or robust optimization frameworks, Silva et al. [29] capture the tradeoff between cost and reliability in a transmission expansion model with demand uncertainty by setting objective coefficients that weight these opposing goals. The authors propose a genetic algorithm solution to find the optimal expansion plan with respect to these weights. The authors allow demand to vary within a range defined by upper and lower bounds. A limitation with the approach proposed in [29] is that it may be difficult in practice to assign appropriate weighting coefficients that allow cost and reliability to be compared in the same units.

An alternative source of uncertainty that has been considered in the transmission expansion literature is the possibility of component failures. Alguacil et al. [1] propose a model for the transmission expansion problem which is robust to intentional line failures. Romero at al. [25] propose a tabu search algorithm to solve the problem of where to add line capacities, as well as generation capacities and spare transformers, in order to ensure that feasible operation is possible in response to a terrorist attack. Choi et al. [5] employ network cut-set constraints to relate probability distributions on the availability of individual components to measures of system-wide resiliency. The authors use this relationship to formulate constraints in a model which seeks a minimum cost transmission expansion which satisfies resiliency criteria. In these works, demand and renewable generation are assumed to be deterministic, and transmission switching is not allowed.

The value of transmission switching has been demonstrated in several papers. Fisher et al. [8], Hedman et al. [13], Hedman et al. [14], Khanabadi et al. [19] and Khodaei and Shahidehpour [20] show how transmission switching might be used to reduce the cost of committing or dispatching generators. Shirokikh et al. [28] present a method of choosing transmission switching actions that minimize generator dispatch costs while ensuring that conditional value at risk constraints are met which would limit the losses in response to contingency event. The authors assume that switching decisions are made prior to the realization of a contingency event and cannot be changed in response to a contingency.

In other works, transmission switching has been shown to be valuable as a corrective action to improve response to a contingency event. In addition to discussing the market implications of transmission switching, Hedman et al. [15] explore how transmission switching might be used to improve resiliency. Li et al. [23] proposed a method for determining the optimal switching actions for the sole purpose of ensuring resilient operation in response to a contingency event.

Several authors have investigated how the transmission expansion problem might be modified to incorporate transmission switching. Khodaei et al. [21] present an algorithm for solving for the minimum cost transmission and generator expansion decisions where transmission switching is employed to reduce dispatch costs. The authors require that the investment solution be feasible for a small set of contingencies, where switching decisions cannot be changed in response to a contingency. The authors use a Benders' decomposition procedure where transmission switching decisions are in the master problem, which is similar to the approach we propose. However, we employ a procedure for dynamically generating switching variables for the master problem on an as-needed basis which allows us to consider a larger set of contingencies, and to additionally consider uncertainty in demand.

Villumsen et al. [32] propose a column generation approach to solving the transmission expansion and switching equipment investment problem when transmission switching is allowed and demands, generator capacities and generator costs are stochastic. The authors in [31] propose a model of the transmission expansion problem when transmission switching is used in response to high wind penetration scenarios.

A problem related to the robust transmission expansion planning problem is that of identifying a worst case event from within the defined uncertainty set for a given expansion solution. This problem is especially interesting when it is assumed that the operator has the ability to react optimally to the event once it has occurred. Neglecting the demand uncertainty and considering only uncertainty in possible line failures, this type of optimization problem is an interdiction problem. Arroyo and Fernández [2], Delgadillo et al. [7] and Zhao and Zeng [37] propose methods for solving this power grid interdiction problem where transmission switching is used as a recovery action. Our proposed approach for solving the oracle described in Section 4.2 extends these previously published methods by identifying a worst-case combination of contingency *and* demand events for a given investment solution. Additionally, the network expansion problem adds a level of complexity to the already challenging power grid interdiction problem since interdiction analysis is a prerequisite to the network optimization problem.

In summary, other authors have considered the transmission expansion planning problem with transmission switching, *or* with uncertainty due to contingency events, *or* uncertainty in demand or renewable generation, but our contribution is to explore novel solution methodologies when all of these complicating factors are considered simultaneously.

3 Problem Definition

We seek an optimal investment solution which determines where new transmission lines should be built and on which lines transmission switching equipment should be installed. The objective is to minimize the total investment cost while ensuring that it is possible to recover from any single transmission line failure and any set of instantaneous demands and renewable generation levels in the defined uncertainty set.

In this section we formally define the robust transmission expansion and switching equipment investment problem. In Section 3.1 we first explain the assumptions that we make in constructing our model. In Section 3.2 we define the deterministic problem, where the transmission line failures (i.e., contingency) and demand vector are given. In Section 3.3 we present the robust counterpart of the deterministic problem, where the contingency and demands/renewable generation vectors may take on any value within their respective uncertainty sets. In Section 3.4 we derive the formulation of the robust counterpart as a linear mixed-integer program (MIP) with an exponential number of constraints. In Section 4, we describe how this MIP formulation can be decomposed and solved via a constraint generation procedure.

3.1 Assumptions

To manage the tradeoff between accuracy and solvability, we make the following assumptions when formulating our model.

- **A set of candidate transmission lines is given.**
  Our investment decisions are binary; for each line in the set of candidate transmission lines, we decide whether or not that line should be built.
- **Transmission switching equipment may be installed on any line.**
  There is a binary decision for each transmission line to determine whether transmission switching equipment should be installed, including both existing and candidate lines. We assume that switching equipment is not

currently installed on any line, but this assumption could easily be modified by fixing the values of certain binary variables.

- **Transmission lines are the only components which may fail.**
  Given the critical nature of transmission lines and the exposure of these lines to weather events, fallen trees, etc., we only consider transmission line failures in contingency events. However, the model presented here may be generalized to include failures of generators as well. Failures in both existing and new transmission lines are considered.
- **Renewable generation is treated as negative demand.**
  Renewable generation is non-dispatchable, meaning that the output cannot be fully controlled by the operator, as availability depends on weather conditions. Typically the only control that the operator has over the renewable generation sources is that excess generation can be curtailed. In our model we assume that renewable generation is always used and never curtailed, but this assumption could easily be relaxed by adding a curtailment decision variable for each renewable generator in the operator's response to a contingency-demand event. The methods presented here are still valid if curtailment is modeled. Because both renewable generation and demand are out of the operator's control, in our model renewable generation is treated the same as negative demand. In the remainder of this paper, the term demand is used to refer to both true demand and renewable generation.
- **Demand values belong to a *box uncertainty set.***
  This uncertainty set on the demand parameters is defined by a lower bound and an upper bound for each node. Our goal is to ensure feasible operation in the event that any demand value within this range is realized. The choice of the demand uncertainty set is influenced by the planners intended planning horizon. A larger planning horizon may necessitate the use of a wider bounds on the demand uncertainty, due to projected population growth or greater uncertainty. The process for determining the appropriate uncertainty set is out of scope for this paper. However, we note that we have chosen to use a box uncertainty set because it is relatively simple. A planner, without access to complex forecasting models may still be able to estimate lower and upper bounds on the power demand/renewable generation at each node based on expert judgement.
- **We use the direct current power flow (DCPF) approximation**.
  We employ a linear approximation of the power flow equations which govern how power flows through the transmission network. This DCPF modeling assumption is commonly used in the academic literature and in industry ([15]).
- **We seek to minimize investment cost, and neglect operational costs.**
  Our goal is primarily to understand where new transmission lines and transmission switching equipment should be installed in order to ensure that feasible operation is possible under all events in our defined uncertainty set. Therefore, in our objective function we include the investment

costs of building new transmission lines or switching equipment, but neglect the operational costs. For shorter planning horizon, investment costs may be considered to be large relative to operational costs, so operational costs are often neglected in the transmission expansion planning literature to ([6], [3], [26]). For longer planning horizons where the operational costs are considered significant, expansion models need to appropriately weight generator dispatch costs for different realizations of demand in the objective function. Due our assumption that a probability distribution on demand is not available and our focus on transmission investment, we have chosen not to model operational costs.

- **Transmission is the dominant limitation, not generator commitments.**
  In our robust formulation, we are primarily interested in making transmission investments that ensure feasible operation for any realization of demand and contingency within the uncertainty set. We assume that during these extreme events, generators are committed appropriately, and lower bounds on generator outputs are not a constraint. Upper bounds are still imposed. This assumption is commonly made in long-term transmission expansion problems ([27]). Transmission is assumed to be the dominant limitation, and so ramping and startup/shutdown constraints on generator operation are relaxed.
- **Only transmission investment decisions are considered.**
  The only investment decisions in our model are decisions about whether to build new transmission lines and transmission switching equipment. We are particularly interested in understanding the interaction between transmission investment and transmission switching decisions, as this can help us understand the value of transmission switching as a recovery action. Power system expansion models often include generation expansion decisions. As future work, this model could be extended to include generation expansion decisions as well.
- **The order in which equipment is installed is not considered.**
  In practice, transmission equipment is installed in a staged way. However, given that we already have included complexities such as uncertainty in demand, renewable generation, and line failures and transmission switching, we have elected not to additionally include a time component in our model, as this would require a major increase in the dimension of the problem. Instead, we seek to solve for the total set of new equipment that would minimize investment costs and satisfy demand within the given uncertainty sets, where the definition of the uncertainty sets be based on a particular planning horizon. The problem of determining the optimal order in which to install this equipment is out of scope for this paper, but would be an interesting problem to consider in future work.

3.2 Deterministic Problem

We first formulate the deterministic problem in which there is no uncertainty in the parameter values; the failure state of all transmission lines is known and the set of nodal demands is known. In this formulation, the binary vector $\bar{c}$ indicates which transmission lines are contained in the given contingency. $\bar{c}_e = 1$ indicates that transmission line e has failed and is *not* available, and $\bar{c}_e = 0$ indicates that transmission line e is available. Additionally, the vector $\bar{d}$ indicates the known demands. Demand $\bar{d}_i$ at node i could either be positive, indicating true demand, or negative, indicating the level of renewable generation at the node. These vectors $\bar{c}$ and $\bar{d}$ will later be allowed to vary within a defined uncertainty set, but for now we assume that these vectors are known.

We note that realistically, the investment decisions must be made before the uncertain contingency and demands are known, and the operating decisions (which we represent by variables y and w) are made in response to the realization of the contingency-demand event. However, in this initial deterministic model where the contingency and demands are known, this distinction of decisions made before and after the realized uncertainty is irrelevant.

The full explicit formulation of the deterministic problem is defined in the Appendix. The compact formulation is defined here using the following vector variable definitions:

- x vector of binary transmission expansion and switching equipment installation decisions. Transmission expansion decisions are made for each line in a set of candidate transmission lines, and transmission switching equipment investment decisions are made for all transmission lines, both existing and candidate.
- y vector of operating decisions including generator outputs, line flows, nodal phase angles, and net power injection at each node.
- w vector of binary transmission switching decisions.

The compact deterministic problem is as follows:

$$\min_{x,y,w} \; b^T x \tag{1a}$$

$$\text{s.t.} \quad Fx \leq f \tag{1b}$$

$$Ay + Bw + Cx \leq h + H\bar{c} \tag{1c}$$

$$Ry = E\bar{d} \tag{1d}$$

$$x, w \text{ binary} \tag{1e}$$

The objective (1a) minimizes the total investment cost of both building new transmission lines and installing new transmission switching equipment. Constraint set (1b) represents constraints on only the investment decisions. These constraints might include a limit on the number of transmission lines that can be built in total or on any particular right-of-way. Constraint set (1c) represents the operational constraints including limits on generator outputs and line capacities, DCPF equations, and power flow conservation. Constraint set (1d) requires that the net power flow out of any particular node is equal to the demand at that node.

3.3 Robust Counterpart Definition

The robust counterpart of the proposed deterministic problem (1) treats the contingency vector c and the demand vector d as uncertain parameters. The vectors c and d are known to belong to uncertainty sets $\mathcal{C}$ and $\mathcal{D}$, respectively. The goal is to solve for a transmission investment solution x such that there exists a nonempty set of feasible recovery operational actions for any $c \in \mathcal{C}$ and $d \in \mathcal{D}$. That is, we seek to identify a transmission investment solution x such that, for any $c \in \mathcal{C}$ and $d \in \mathcal{D}$, there exists at least one set of feasible operations decisions y and switching decisions w. More formally, the robust formulation is stated as follows.

$$
\begin{aligned}
\min_{x} \quad & b^T x && \text{(2a)}\\
\text{s.t.} \quad & Fx \leq f && \text{(2b)}\\
& \mathcal{F}(x, c, d) \text{ nonempty } \forall c \in \mathcal{C}, d \in \mathcal{D} && \text{(2c)}\\
& x \text{ binary} && \text{(2d)}
\end{aligned}
$$

where

$$
\mathcal{F}(x, c, d) = \begin{cases} Ay + Bw \leq h + Hc - Cx \\ Ry = Ed \\ w \text{ binary} \\ y \text{ unbounded} \end{cases}
$$

We assume that the uncertainty set of contingencies $\mathcal{C}$ contains all contingencies of size 1 or 0. That is, $\mathcal{C} = \{c \in \{0, 1\}^{|\mathcal{E}|} : e^T c \leq 1\}$, where $\mathcal{E}$ represents all existing and candidate transmission lines and e is an appropriately sized unit vector.

For the demand uncertainty set $\mathcal{D}$, we use a box uncertainty set. That is, the demand (and/or renewable generation) at each node is allowed to vary within predefined upper and lower bounds. Let $\mathcal{N}$ be the set of all nodes, and L_i and U_i be the lower and upper bounds on the demand for node i, respectively. Thus, $\mathcal{D} = \left\{d \in \mathbb{R}^{|\mathcal{N}|} : L_i \leq d_i \leq U_i \ \forall i \in \mathcal{N}\right\}$.

In its current form, (2) is difficult to solve directly. Thus, in Section 3.4 we show that (2) can be formulated as a single-level MIP with an exponential number of variables and constraints.

3.4 Robust Counterpart Reformulation

Given a particular investment solution x, contingency c and demand vector d, the operator may choose a set of operational decisions y and a switching configuration w to best respond to the particular contingency-demand event. However, the existence of binary switching variables w makes the robust formulation much more complex, so for the moment let us assume that the switching configuration w is fixed *a priori.* The operator then must choose a set of operational decisions y that are feasible for the following *fixed-switching recovery problem*:

$$S^P(x,w,c,d) = \min_{y} \ 0 \tag{3a}$$
$$\text{s.t.} \quad Ay \leq h + Hc - Bw - Cx \qquad (\phi) \tag{3b}$$
$$Ry = Ed \qquad (\eta) \tag{3c}$$

The dual of (3) is as follows:

$$S^D(x,w,c,d) = \max_{\phi,\eta} \phi^T(h + Hc - Bw - Cx) + \eta^T Ed \tag{4a}$$
$$\text{s.t.} \quad A^T\phi + R^T\eta = 0 \qquad (y) \tag{4b}$$
$$\phi \leq 0 \tag{4c}$$

The solution $\phi = 0, \eta = 0$ is feasible for (4) for any A and R, thus (4) is feasible for any inputs x, w, c and d.

If (4) has an optimal objective value equal to 0, then a feasible solution exists for the fixed-switching recovery problem (3). Otherwise, (4) is unbounded, and (3) does not have a feasible solution.

For a given investment solution x and switching configuration w, formulation (4) can be modified to find contingency and demand vectors that make the fixed-switching recovery problem infeasible by letting c and d become variables which may take any value within their respective uncertainty sets. If c and d become variables, (4) becomes the following optimization problem.

$$R(x,w) = \max_{d,c,\phi,\eta} \ \phi^T(h + Hc - Bw - Cx) + \eta^T Ed \tag{5a}$$
$$\text{s.t.} \quad A^T\phi + R^T\eta = 0 \tag{5b}$$
$$\phi \leq 0 \tag{5c}$$
$$c \in \mathcal{C} \tag{5d}$$
$$d \in \mathcal{D} \tag{5e}$$

If the optimal objective value $R(x,w)$ is infinity, then a contingency-demand pair has been identified which causes (3) to be infeasible. However, (5) contains bilinear terms in the objective function and thus in its current form, it is not easy to solve.

Since the uncertainty set $\mathcal{D}$ can be expressed with a linear system of constraints which are disjoint with the other constraints (5b)-(5d), the optimal solution for d for (5) will be an extreme point of the polyhedron $\mathcal{D}$ ([17]). Furthermore, the optimal solution for (ϕ, η) for (5) must be an extreme point or extreme ray of the feasible region defined by constraints (5b)-(5c), for the same reason.

Let ext($\mathcal{D}$) represent the set of extreme points of the polyhedron $\mathcal{D}$. Additionally, let $\mathcal{V}$ represent the set of extreme rays of the feasible region of (4), and let $\mathcal{X}$ represent the set of extreme points of the feasible region of (4). Given that the constraints (5b)-(5c), (5d) and (5e) are disjoint from each other in the sense that they do not contain any common variables, and that the sets $\mathcal{X}$, $\mathcal{V}$, ext($\mathcal{D}$) and $\mathcal{C}$ all contain a finite number of elements, (5) can be rewritten as the following combinatorial optimization problem:

$$R(x, w) = \max_{(\phi,\eta)\in\mathcal{X}\cup\mathcal{V},\ d\in\text{ext}(\mathcal{D}),\ c\in\mathcal{C}} \{\phi^T(h + Hc - Bw - Cx) + \eta^T Ed\} \tag{6}$$

If the optimal objective value $R(x, w) = 0$, then there exists a feasible solution to the fixed-switching recovery problem (3) for all $d \in \text{ext}(\mathcal{D})$ and $c \in \mathcal{C}$ for the particular investment decisions x and switching recovery decisions w.

However, what we are really interested in is whether, for all $d \in \text{ext}(\mathcal{D})$ and $c \in \mathcal{C}$, there exists a feasible solution to the recovery problem where switching is not fixed but allowed to be chosen in response to each (c, d) pair. Or, put another way, whether there exists at least one switching configuration for each contingency-demand pair for which there exists a feasible solution to the fixed-switching recovery problem.

Let $i(c)$ be a function that maps the contingency c to its corresponding index in the set $\mathcal{C}$. That is, for any $c \in \mathcal{C}$, $i(c) \in \{1, 2, \ldots, |\mathcal{C}|\}$. Similarly, let $j(d)$ be a function that maps a demand vector d which is an extreme point of the set $\mathcal{D}$ to its corresponding index in ext($\mathcal{D}$). That is, for any $d \in \text{ext}(\mathcal{D})$, $j(d) \in \{1, 2, \ldots, |\text{ext}(\mathcal{D})|\}$.

For a given x, the requirement that there must exist a switching configuration $w^{i(c),j(d)}$ for all $d \in \text{ext}(\mathcal{D})$ and $c \in \mathcal{C}$ that enables the fixed-switching recovery problem to be feasible can be expressed as follows:

$$\exists w^{i(c),j(d)} \forall c \in \mathcal{C}, d \in \text{ext}(\mathcal{D}) : R(x, w^{i(c),j(d)}) = 0$$

Thus, the formulation of the robust counterpart of the deterministic transmission expansion problem (1) is as follows.

$$\min_{x,w} \ b^T x \tag{7a}$$

$$\text{s.t.} \quad Fx \leq f \tag{7b}$$

$$\phi^T(h + Hc - Bw^{i(c),j(d)} - Cx) + \eta^T Ed \leq 0 \tag{7c}$$

$$\forall (\phi, \eta) \in \mathcal{X} \cup \mathcal{V},\ d \in \text{ext}(\mathcal{D}),\ c \in \mathcal{C}$$

$$x \text{ binary} \tag{7d}$$

$$w^{i(c),j(d)} \text{ binary} \quad \forall d \in \text{ext}(\mathcal{D}),\ c \in \mathcal{C} \tag{7e}$$

Constraint (7c) enforces that for any feasible investment solution x, there must exist a switching configuration $w^{i(c),j(d)}$ such that the optimal objective function of the combinatorial optimization program (6) is less than or equal to 0. This requirement ensures the existence of a feasible recovery solution in response to every event in the uncertainty set $\mathcal{C} \times \text{ext}(\mathcal{D})$.

4 Algorithmic Development

Formulation (7) is a linear MIP which can be used to find an investment solution x that minimizes investment cost and ensures that feasible operation is possible in response to any event in the defined uncertainty set. However, (7) contains an exponential number of constraints and variables, as the sets $\mathcal{X} \cup \mathcal{V}$ and $\text{ext}(\mathcal{D})$ both contains an exponential number of elements. Thus, to solve this MIP in practice, we employ a decomposition procedure.

4.1 Decomposition

To decompose (7), the constraint set containing an exponential number of constraints, (7c), is relaxed and violated constraints from the set (7c) are iteratively generated, as needed, until a feasible investment solution x is identified.

Before proceeding with our decomposition approach, we briefly discuss the challenges of solving the transmission expansion planning problem (2) by standard decomposition methods such as Benders decomposition. Two factors contribute to the computational difficulty of solving (2) or its equivalent single-level reformulation (7). First, although the uncertainty set $\mathcal{C} \times \text{ext}(\mathcal{D})$ is finite it is extremely large. As such, it is impractical to screen all elements of this uncertainty set explicitly, even if checking feasibility only involves solving a relative straight forward mixed-integer program. Therefore, instead of considering each contingency-demand scenario explicitly we proposed an Oracle based approach where the worst-case contingency-demand scenario is identified by solving a bilevel integer program. Second, the binary switching variables w destroy the convexity of the subproblem and the reformulation of the separation oracle, which is key to any solution approach based on Benders Decomposition. Thus, recent solution approaches for robust power grid optimization proposed in ([17]) and ([18]), which are based on Benders decomposition, are not directly applicable.

4.1.1 Switching Variable Generation

Formulation (7) has the form of a master problem in a two-stage stochastic program in which the first stage variables are x and w, and the second stage problem is (3) with second stage variables y. The set of scenarios is the set of all contingency-demand pairs in the set $\mathcal{C} \times \text{ext}(\mathcal{D})$. Constraint set (7c) represents the set of Benders' feasibility cuts corresponding to all extreme points and extreme rays of the feasible region of the dual of the second stage problem for all scenarios.

We note that the switching variables are naturally second stage variables, because in practice the switching decisions can be chosen in response to particular contingency-demand event. However, we have effectively moved the switching variables into the first stage to alleviate the difficulty of solving a problem with second stage integer variables.

One challenge with first stage switching decisions is that it results in a very large number of binary variables in the master problem. There exists a switching vector $w^{i(c),j(d)}$ in the master problem for every contingency-demand pair (c, d). As previously mentioned, the set $\mathcal{C} \times \text{ext}(\mathcal{D})$ contains an exponential number of elements, and thus there will exist a very large number of switching variables in the master problem even for relatively small systems.

To address this challenge, we propose that switching variables be generated iteratively as needed as (7) is solved via a constraint generation procedure. The feasibility cuts in the set (7c) are initially relaxed and are then incrementally added as violations are identified. Any first stage variables that are not contained in any constraints can effectively be ignored. Switching variables are only contained in the constraints (7c), so initially all switching variables can be ignored. As violated constraints from (7c) are identified iteratively, each of which corresponds to a particular contingency-demand pair (c, d), we generate the corresponding switching variables $w^{i(c),j(d)}$. Thus, the number of switching variables effectively in the master problem grows gradually as cutting planes are generated for the master problem.

In practice, we have found that switching variables are generated for only a small subset of all contingency-demand pairs. This observation will be further discussed in Section 5.

4.1.2 Oracle Motivation

A problem with the structure of (7) would traditionally be solved with Benders' decomposition in which feasibility cuts in the set (7c) are generated by solving subproblems (3) for every contingency-demand pair in each iteration. Given that the set of all demand extreme points $\text{ext}(\mathcal{D})$ contains an exponential number of elements, it would take an impractically long time to solve a subproblem for every $(c, d) \in \mathcal{C} \times \text{ext}(\mathcal{D})$ in each iteration.

To address this challenge, we develop an *oracle*. The goal of the oracle is to identify a contingency-demand pair that does *not* have a feasible recovery solution for the current investment solution x, even with the best possible switching configuration. The development of the oracle will be explained further in Section 4.2, but for now let us assume that such an oracle exists. This oracle eliminates the need to explicitly screen all contingencies and demand pairs in the uncertainty set in order to identify an unsurvivable contingency-demand pair.

4.1.3 Cutting Plane Algorithm

Assuming that there exists an oracle for identifying unsurvivable contingency-demand pairs, the proposed algorithm for finding the minimum cost robust investment solution proceeds as follows. An illustration of the algorithm is presented in Figure 1.

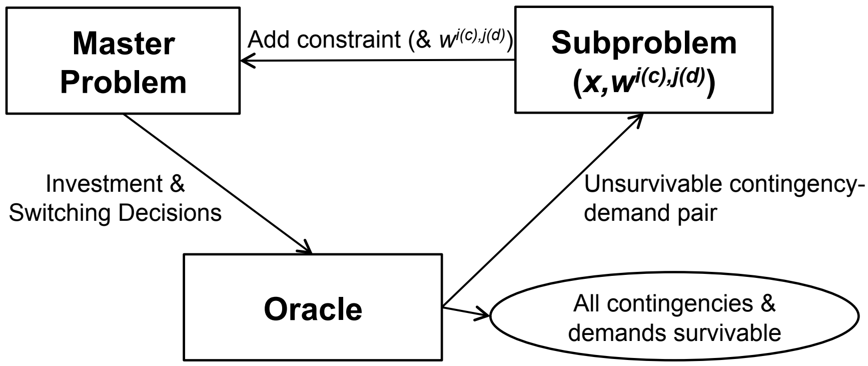

Fig. 1 Complete Algorithm Overview

In each iteration, the master problem is solved. Initially, the master problem is (7) where all constraints in set (7c) are relaxed, and all switching variables w are ignored. The oracle is called to identify an unsurvivable contingency-demand pair $(\bar{c}, \bar{d})$. A subproblem for this $(\bar{c}, \bar{d})$ pair is solved, and the dual subproblem solution $(\bar{\phi}, \bar{\eta})$ is used to generate a feasibility cut for the master problem. The form of the feasibility cut is as follows:

$$\bar{\phi}^T (h + H\bar{c} - Bw^{i(c),j(d)} - Cx) + \bar{\eta}^T E\bar{d} \leq 0$$

As feasibility cuts in the set (7c) are generated for the master problem, corresponding sets of switching variables are added as well. The procedure of generating feasibility cuts repeats until the oracle identifies that all contingencies and demands in the uncertainty set are survivable for the current investment solution x.

4.2 Oracle Development

The role of the oracle is to identify a contingency-demand pair for which feasible operation is not possible given the current investment solution x, even when transmission switching is available as a recovery action. If no such unsurvivable contingency-demand pair exists, the oracle should return a certification to indicate that the current investment decision x is optimal. This type of problem can be thought of in terms of a fictional adversary who seeks to identify a transmission line to disrupt and a particular demand scenario whose combination would maximize damage.

Recall that the optimal solution to (5) identifies a contingency-demand pair that would be unsurvivable for a given set of investment decisions x *if the recovery switching configuration were fixed.* If (c, d) is unsurvivable with this particular fixed switching configuration, this is *not* a certification that (c, d) would also be unsurvivable under a different, better switching configuration. However, the optimal solution (c, d) to (5) is a good candidate for unsurvivability. We use this optimization problem (5) within an iterative constraint generation routine which alternately identifies (c, d) pairs which are candidates for unsurvivability, and verifies whether a given (c, d) pair is actually unsurvivable when any switching configuration is allowed as a recovery action.

Before this constraint generation routine is presented, we first present a reformulation of (5) which eliminates bilinear terms in the objective to become a linear MIP.

4.2.1 Bilinear reformulation

To transform formulation (5) into a linear MIP, the two bilinear terms in the objective function, $\phi^T Hc$ and $\eta^T Ed$, must be linearized.

The linearization of the first term is fairly simple because the contingency variables are binary, and so the bilinear term is a product of a binary variable and a continuous variable. There exist standard methods for linearizing this type of bilinear term ([12]).

A new set of auxiliary variables can be defined, γ_e, to represents the bilinear quantity $(\phi^T H)_e\ c_e$. In the objective, the bilinear term $\phi^T Hc$ is replaced with the linear term $e^T \gamma$, where e is an appropriately sized unit vector. To enforce the relationship between γ_e and the original bilinear terms, the following set of constraints is added for each transmission line $e \in \mathcal{E}$.

$$\gamma_e \le Mc_e \tag{8a}$$
$$\gamma_e \ge -Mc_e \tag{8b}$$
$$\gamma_e \le (\phi^T H)_e + M(1 - c_e) \tag{8c}$$
$$\gamma_e \ge (\phi^T H)_e - M(1 - c_e) \tag{8d}$$

Let M be a parameter defined such that $M \ge \max_{(\phi,\eta)\in\mathcal{X}\cup\mathcal{V},e\in\mathcal{E}}\{-(\phi^T H)_e, (\phi^T H)_e\}$. Constraints (8a)-(8b) enforce that $\gamma_e = 0$ when $c_e = 0$, and constraints (8c)-(8d) enforce that $\gamma_e = (\phi^T H)_e$ when $c_e = 1$. The set of equations (8a)-(8d) effectively enforce the original bilinear relationship that $\gamma_e = (\phi^T H)_e\ c_e$ for each $e \in \mathcal{E}$. In compact form, let the constraints (8a)-(8d) for all $e \in \mathcal{E}$ be represented by the constraint $G\gamma \le g + Jc + Q\phi$.

The linearization of the second term $\eta^T Ed$ is more complex, as the demand d_i is a continuous variable which may take on any value within the specified upper and lower bounds. We propose an alternative representation of the demand d_i in terms of binary variables.

As discussed in Section 3.3, $\mathcal{D} = \left\{d \in \mathbb{R}^{|\mathcal{N}|} : L_i \leq d_i \leq U_i \; \forall i \in \mathcal{N}\right\}$, and the optimal solution for d to the optimization problem (5) will always be one of the extreme points of the polyhedral uncertainty set $\mathcal{D}$. Given the definition of the box uncertainty set, at an extreme point of $\mathcal{D}$, the demand d_i is either equal to its upper bound U_i or equal to its lower bound L_i. Thus, at any extreme point, the demand d_i can be represented in terms of a binary variable z_i. Let z_i equal 1 if $d_i = U_i$, or equal 0 if $d_i = L_i$. Thus, the demand d_i at an extreme point of $\mathcal{D}$ can be expressed as follows:

$$d_i = L_i + (U_i - L_i)z_i; \;\; z \text{ binary}$$

In the objective of the bilevel program, the demand variable d_i is multiplied by $(E^T\eta)_i$. For each element i, the term in the objective is rewritten as:

$$(E^T\eta)_i \; d_i = (E^T\eta)_i \; L_i + (E^T\eta)_i \; (U_i - L_i)z_i$$

Note that this expression contains bilinear terms, as $(E^T\eta)_i$ is a continuous variable and z_i is a binary variable. However, as this bilinear term is the product of a binary variable and a continuous variable, it can be linearized in the same way as was $(\phi^T H)_e \; c_e$. Let λ_i be the auxiliary variable which represents the bilinear term $(E^T\eta)_i z_i$. Let constraints analogous to (8a)-(8d) enforce the relationship that $\lambda_i = (E^T\eta)_i z_i$, and let the compact representation of these constraints be $S\lambda \leq s + Tz + V\eta$.

Let the term $\eta^T Ed$ in the objective (5a) be replaced by $\eta^T EL + (U-L)^T\lambda$, which represents the linearized expression.
Thus, the nonlinear optimization problem (5) can be reformulated as a linear MIP as follows:

$$R(x,w) = \max_{\phi,\eta,c,\gamma,z,\lambda} \; \phi^T h + e^T\gamma + \phi^T(-Bw) + \phi^T(-Cx) + \eta^T EL + (U-L)^T\lambda \tag{9a}$$

$$\begin{aligned}
\text{s.t.} \quad & H^T\phi + R^T\eta = 0^T && \text{(9b)}\\
& G\gamma \leq g + Jc + \phi && \text{(9c)}\\
& S\lambda \leq s + Tz + V\eta && \text{(9d)}\\
& \phi \leq 0 && \text{(9e)}\\
& e^T c \leq 1 && \text{(9f)}\\
& c, z \text{ binary} && \text{(9g)}
\end{aligned}$$

This linearized program (9) can be solved directly to identify a contingency-demand pair for which feasible recourse is not possible for the given investment x and a fixed switching configuration w.

There exist several other special types of uncertainty sets for which the extreme points can be expressed in terms of binary variables. Other authors have used this type of uncertainty set representation in robust power system optimization problems for a polyhedral uncertainty set ([18]), a budget uncertainty set ([17]) and multiple budget uncertainty sets ([38]). For these types of uncertainty sets, the basic procedure presented in this section for developing the oracle is applicable.

4.2.2 Iterative Oracle Routine

The reformulated program (9) can identify a contingency-demand pair which is unsurvivable for a given investment decision x when recovery switching actions are fixed to a given w. However, what we are more interested in is a contingency-demand pair which is unsurvivable when the set of switching actions w is not fixed *a priori*, but is allowed to be chosen in response to particular (c, d) event. An iterative routine is proposed to identify a contingency-demand pair that unsurvivable for all possible switching configurations.

The routine iterates between solving an upper level problem and a lower level problem. The upper level problem identifies a contingency-demand pair (c, d) which is unsurvivable for the current investment solution for a given fixed switching configuration. That (c, d) pair is passed to the lower level to definitively determine whether a particular (c, d) pair is unsurvivable when *any* switching configuration may be chosen in response to that event. The routine for solving the oracle is illustrated in Figure 2.

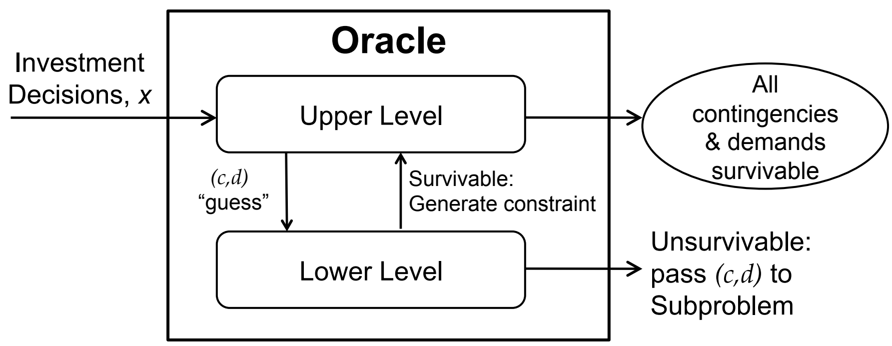

Fig. 2 Oracle Routine

More specifically, the routine for solving the oracle proceeds as follows. Let $\bar{x}$ be the current investment solution.

First, the initial upper level problem (9) is formulated where the switching vector is fixed to $w = 0$ (i.e., no switching). If the optimal objective value of the upper level problem is unbounded, a contingency-demand pair $(\bar{c}, \bar{d})$ has been identified which is unsurvivable when switching is fixed to $w = 0$. This $(\bar{c}, \bar{d})$ pair is passed to the lower level problem to check whether there exists a different switching configuration that would enable survivability.

The lower level problem is formulated as follows.

$$S(x,c,d) = \min_{y,w} \; 0 \tag{10a}$$

$$\text{s.t.} \quad Ay + Bw \leq h + Hc - Cx \tag{10b}$$

$$Ry = Ed \tag{10c}$$

$$w \text{ binary} \tag{10d}$$

If the lower level problem is infeasible, then there does not exist any switching configuration that would allow there to exist a feasible recovery solution for this $(\bar{c}, \bar{d})$, meaning that $(\bar{c}, \bar{d})$ is unsurvivable. The oracle routine can be exited, and this $(\bar{c}, \bar{d})$ pair can then be passed to the subproblem (3).

Otherwise, the lower level problem is feasible, indicating that there exists a switching configuration that enables survivability. A constraint is generated for the upper level problem to make the current contingency-demand solution infeasible. Let the optimal switching configuration in the feasible lower level solution be $\hat{w}$. The constraint added to the upper level problem requires that the dual objective value $S^D(\bar{x}, \hat{w}, \bar{c}, \bar{d})$ be greater than 0. A dual objective that is greater than 0 indicates that the fixed-switching recovery problem is infeasible. An unsurvivable (c,d) pair must be unsurvivable for all possible switching configurations, so requiring that the dual objective is greater than 0 for any particular switching configuration is valid.

Let ϵ be a very small value which is the threshold at which a value is considered to be "greater than 0". The constraint generated for the upper level problem is as follows:

$$\phi^T h + e^T \gamma + \phi^T(-B\hat{w}) + \phi^T(Cx) + \eta^T EL + (U-L)^T \lambda \geq \epsilon$$

The upper level problem is solved again, and in the next iteration a new contingency-demand pair will be identified. The procedure repeats until the lower level becomes infeasible, indicating that an unsurvivable (c,d) pair has been found, or the upper level becomes infeasible or has an optimal objective value equal to 0, indicating that there does not exist an unsurvivable (c,d) pair in the uncertainty set.

4.3 Implementation

An overview of the cutting plane algorithm was described in Section 4.1.3. In Section 4.3.1 we discuss two details of the implementation which improve the rate of convergence, and in Section 4.3.2 the complete algorithm including these implementation details will be defined.

4.3.1 Implementation Details

One implementation details contains the inputs to the subproblem (3), x, w, c, d. The contingency c and demand d are set according to the unsurvivable contingency-demand pair identified by the oracle. The investment x is set according to the master problem solution, and the switching vector w may or may not be set according to the master problem solution, depending upon whether the switching vector $w^{i(c),j(d)}$ exists in the master problem. If $w^{i(c),j(d)}$ exists in the master problem, then to guarantee convergence, w must be set equal to the master problem solution for $w^{i(c),j(d)}$. However, if $w^{i(c),j(d)}$ has not been added to the master problem, then any arbitrary binary vector may be set which is feasible for the current investment solution x. That is, the switching configuration chosen may only switch lines out which have switching equipment installed on them according to the investment solution x. For simplicity, we choose to set $w = 0$.

The second issue concerns the manner in which unsurvivable contingency-demand pairs are identified. To ensure feasibility of a given (c, d) pair, multiple feasibility cuts are often necessary. Thus, rather than calling the oracle to identify an unsurvivable (c, d) pair in every iteration, we suggest that a *Critical* (c, d) *List* instead be checked for unsurvivable (c, d) pairs. This critical list is a list of all (c, d) pairs that have previously been identified as unsurvivable by the oracle, which are good candidates for unsurvivability.

The procedure for identifying unsurvivable contingency-demand pairs from the critical list is as follows. Given the current investment solution x, for each contingency-demand pair in the list, the with-switching recovery problem (10) is solved. If (10) is infeasible for any (c, d) pair, then an unsurvivable (c, d) pair has been identified. If (10) is feasible for all contingency-demand pairs in the critical list, the oracle is called to identify an unsurvivable contingency-demand pair, if one exists.

In our computational tests, we have found that there tends to exist a small set of "dominant" contingency-demand pairs, in the sense that once sufficient investments are made to ensure the survivability of these pairs, all other pairs are also survivable. Thus, checking this Critical List seems to be an efficient way to identify unsurvivable contingency-demand pairs.

4.3.2 Complete Algorithm

Using these implementation details, the complete cutting plane algorithm is presented here.

1. Solve the initial master problem, which is (7) where all constraints in the set (7c) are relaxed, and there are no switching variables w. Get the optimal solution $\hat{x}$.
2. *Oracle Routine*
   (a) Set $w = 0$ and solve the initial upper level problem (9) for the optimal solution $(\hat{\phi}, \hat{\eta}, \hat{c}, \hat{\gamma}, \hat{z}, \hat{\lambda})$. If $R(x, 0) \leq 0$, then exit with the optimal investment solution x, which is robust to all contingency-demand pairs in the uncertainty set. Otherwise, use the optimal solution to generate the candidate contingency-demand pair $(\hat{c}, \hat{d})$, where $\hat{d} = L + (U - L)^T \hat{z}$.

(b) Pass $(\hat{x}, \hat{c}, \hat{d})$ to the lower level problem (10) and solve. If (10) is feasible, then $(\hat{c}, \hat{d})$ is survivable. Let the optimal switching configuration be $\hat{w}$. Otherwise, go to step 3.
(c) Add the following constraint to the upper level problem.

$$\phi^T h + e^T \gamma + \phi^T(-B\hat{w}) + \phi^T(C\hat{x}) + \eta^T EL + (U - L)^T \lambda \geq \epsilon$$

(d) Solve the upper level problem. If the upper level problem is infeasible or if the optimal objective value is 0, then exit with the optimal investment solution $\hat{x}$, which is robust to all contingency-demand pairs in the uncertainty set. Otherwise, use the optimal solution $(\hat{\phi}, \hat{\eta}, \hat{c}, \hat{\gamma}, \hat{z}, \hat{\lambda})$ to generate the candidate $(\hat{c}, \hat{d})$ pair. Continue to step 2b.

3. Add $(\hat{c}, \hat{d})$ to the critical list if it does not already exist.
4. If the switching vector $w^{i(\hat{c}),j(\hat{d})}$ exists in the master problem, pass $(\hat{x}, \hat{w}^{i(\hat{c}),j(\hat{d})}, \hat{c}, \hat{d})$ to the fixed-switching recovery dual subproblem (4) and solve for the optimal solution $(\tilde{\phi}, \tilde{\eta})$.
5. Otherwise, if the switching vector $w^{i(\hat{c}),j(\hat{d})}$ does not yet exist in the master problem, pass $(x, 0, \hat{c}, \hat{d})$ to the fixed-switching recovery dual subproblem (4) and solve for the optimal solution $(\tilde{\phi}, \tilde{\eta})$.
6. Generate the following feasibility cut for the master problem:

$$\tilde{\phi}^T(h + H\hat{c} - Bw^{i(\hat{c}),j(\hat{d})} - Cx) + \tilde{\eta}^T E\hat{d} \leq 0$$

If $w^{i(\hat{c}),j(\hat{d})}$ did not previously exist in the master problem, it is now added to the master problem.
7. Solve the master problem for the optimal solution $(\hat{x}, \hat{w})$.
8. For each (c, d) pair in the critical list, solve the with-switching recovery problem (10). If (10) is infeasible for any $(\hat{c}, \hat{d})$, stop looping through the critical list. Pass $(\hat{x}, \hat{w}^{i(\hat{c}),j(\hat{d})}, \hat{c}, \hat{d})$ to the dual subproblem (4). Let the optimal dual solution be $(\tilde{\phi}, \tilde{\eta})$. Go to step 6.
Otherwise, if (10) is feasible for all (c, d) pairs in the critical list, go to step 2.

4.3.3 Convergence

We note that the algorithm described in Section 4.3.2 is guaranteed to terminate in a finite number of iterations with a globally optimal solution. Recall that (7), which is equivalent to the robust transmission expansion problem (2), is a linear MIP. In the simplest case, bypassing the Oracle Routine, this is equivalent to a traditional Benders' decomposition approach, with first stage variables x and w and second stage variables y. In the worst case, Benders' terminates after cuts have been generated for each extreme point and extreme ray of the dual subproblem. By adding in the Oracle Routine, we simply reduce the time required to generate the Benders' cuts.

For the Oracle Routine itself, the routine is guarenteed to exit with either an unsurvivable (c, d) pair or the certification that all contingencies and demands are survivable because there are a finite number of contingency-demand pairs and a finite number of switching configurations. In the worst case, a constraint could be generated for the Upper Level problem for every single (c, d) and switching configuration. Because the oracle is guaranteed to identify an unsurvivable (c, d) pair, if one exists, then in terms of convergence of the cutting plane algorithm, it is effectively the same as checking through the list of all scenarios (which here are (c, d) pairs), as one would do in traditional Benders'.

5 Computational Results

The proposed algorithm was implemented in C++ using CPLEX v12.4 Concert Technology. Our computational results were performed on a computer with 4GB RAM and a 2.3 GHz processor. In all computational experiments, the algorithm was run until the optimal solution was identified, where the default parameters regarding the optimality gap, etc. in CPLEX were used.

Results for three different test cases are presented here. The original sources for these test cases ([9], [10], [11]) include descriptions of the topology and system characteristics, but do not however include sets of candidate lines. We use sets of candidate lines from relevant transmission expansion literature. For the IEEE24 test system and Garver test system, candidate lines from [1] are used. The set of candidate lines and the capacities for the existing lines for the IEEE14 test case are from [34]. Costs of installing transmission switching equipment was not available in the references, so we chose switching costs to approximately match the relative cost of switching equipment and new transmission lines defined in [31]. The essential characteristics of the test cases are summarized in Table 1

Test Case	# Nodes	# Loads	# Generators	# Existing Lines	# Candidate Lines
Garver6	6	5	3	6	39
IEEE14	14	11	5	20	10
IEEE24	24	17	32	35	10

Table 1 Garver6, IEEE14 and IEEE24 System Characteristics

The runtime ranges for each of the test cases are shown in Table 2 for a variety of different demand uncertainty sets, as defined later in this section. Additionally, the column titled "#(c,d) pairs considered" in Table 2 refers to the number of contingency-demand pairs explicitly considered when solving the algorithm over the same range of demand uncertainty sets. This refers to the number of unique contingency-demand pairs identified by the oracle while solving the algorithm, whose corresponding switching variables are effectively added to the master problem. The column "Total # of (c,d) pairs" indicates the number of elements in the set $\mathcal{C} \times \text{ext}(\mathcal{D})$, which is determined by the number of existing and candidate transmission lines and the number of demand nodes. Our intention is to demonstrate the order of magnitude of the number of contingency-demand pairs that are explicitly considered while solving the algorithm relative to the total number in the uncertainty set. As shown in Table 2, for these test cases only a handful of contingency-demand pairs were explicitly considered, despite the thousands or millions of contingency-demand pairs in the uncertainty set.

Test Case	Run Time	# (c,d) pairs considered	Total # (c,d) pairs
Garver6	17-94 sec.	7-11	1.4E3
IEEE14	1-9 sec.	1-2	6.1E4
IEEE24	13-99 sec.	2-5	5.9E6

Table 2 Run Times and Performance Metrics

We note that for these three test instance, the larger networks do not necessarily have longer run times. The number of candidate lines, and the total number of new investments that must be made to ensure resiliency, seem to be a more important indicator for how long the algorithm will take to converge. If more investments are needed, the algorithm will require more iterations. More conservative demand uncertainty sets require more investments, so longer run times are correlated with wider demand uncertainty sets. We would expect that for a problem where the demand uncertainty set has been defined widely to represent the needs over a long planning horizon, the algorithm would run more slowly, whereas narrower demand uncertainty sets representing shorter planning horizons could be solved more quickly. For these test instances, the algorithm's run time was dominated by the time required to solve the master problem.

As mentioned in Section 4.3.3, with our proposed algorithm it is theoretically possible that in the worst case it would be necessary to explore an exponential number of contingency-demand pairs. However, we demonstrate that for these small test cases, with our proposed algorithm it is only necessary to explore a small fraction of all contingency-demand pairs. While the algorithmic performance strongly depends on the particular problem parameters, we expect that this property holds for larger test instances as well. We have used a serial implementation of the proposed algorithm, but a parallel implementation may useful for solving larger problems.

For the Garver 6 bus test instance, we explore how the use of transmission switching as a recovery action changes the investment solution. Figure 3 represents the optimal investment solution when transmission switching is *not* an allowable recovery action, and Figure 4 represents the optimal investment solution when transmission switching is allowed. The dashed lines represent new transmission lines. In Figure 4, the circuit breaker image on the transmission line between nodes 2 and 3 represents new switching equipment. The solutions illustrated in Figures 3 and 4 share many of the same investments. However, the optimal cost with switching is \$184 compared to \$200, when transmission switching is not allowed. This is due to the fact that when transmission switching is allowed, 1 fewer transmission lines are built, and transmission switching equipment is built on one line. The transmission switching equipment is much cheaper than building a new transmission line.

Note that by switching line (2-3) out, the cycle between nodes 1, 2, 3 and 5 is broken. Transmission switching is most likely to be useful in transmission networks which contain cycles. In a network which more resembles a tree or a line, there is a lot of flexibility to find phase angle values that would support whatever power flow patterns are desired. However, in a dense network with cycles, the DCPF constraints are likely to be limiting, as phase angle values are more constrained. Thus, these are the systems where transmission switching is most likely to be useful.

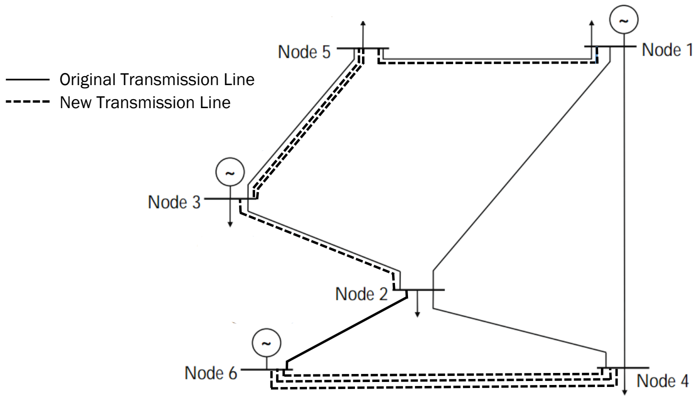

Fig. 3 Optimal Investment Solution without Transmission Switching

In an effort to explore how the conservatism of the defined demand uncertainty set impacts the optimal cost, for the IEEE14 and IEEE24 bus test cases, we have fixed the lower bound on the demand uncertainty set and scaled the upper bound. The lower bound is set equal to 70% of the nominal demand. The upper bound is set equal to the nominal demand times a scaling factor. High scaling levels for the demand upper bound represent an increased level of conservatism in the defined uncertainty set. Figure 5 represents the optimal investment cost for different scaling factors for the demand upper bound for the IEEE24 test case. Similarly, Figure 5 represents the same quantities for the IEEE14 test case.

We note that for both of these test cases, the cost of the optimal investment solution is lower when switching is allowed as a recovery action than when transmission switching is not employed as a recovery action. Essentially,

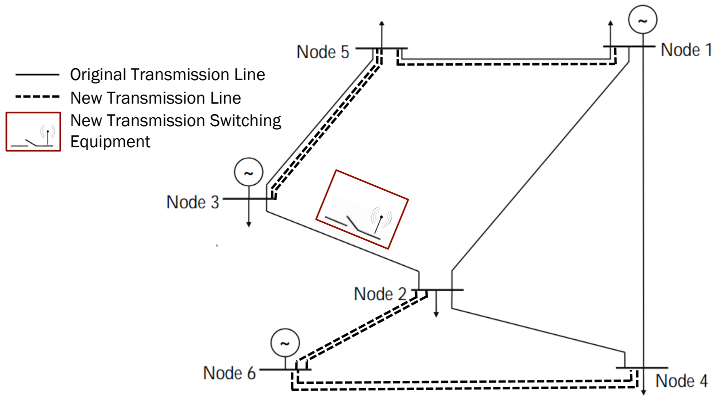

Fig. 4 Optimal Investment Solution with Transmission Switching

the flexibility introduced by transmission switching allows the same level of resiliency to be achieved by installing new switching equipment rather than new transmission lines, as the cost of the switching equipment is small relative to the cost of new transmission lines. In Figure 6, when the demand upper bound is fixed to 100%, the optimal cost with transmission switching is shown, but without transmission switching, a feasible solution is not possible. Thus, it some instances allowing transmission switching may allow a level of resiliency to be attained that would not be possible under any investment solution when transmission switching is not employed.

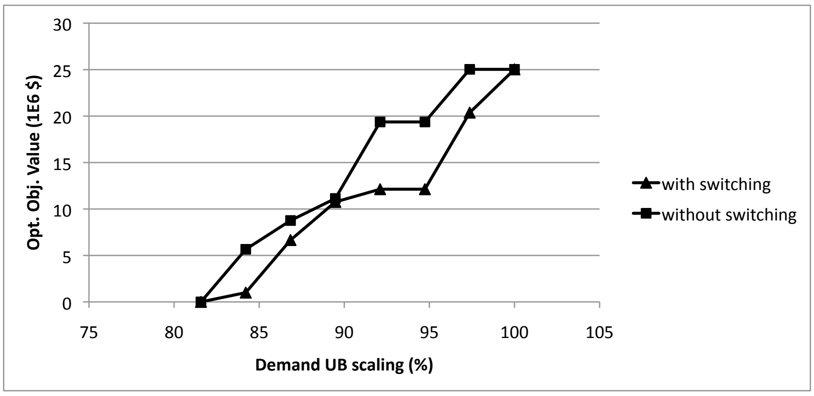

Fig. 5 Optimal Investment Cost for IEEE24 Test Case with Scaled Demand

We note that another way to visualize optimal cost as a function of relative congestion is to vary the transmission line capacities. In the IEEE24 test case, the existing transmission lines are divided among “low” and “high” capacity

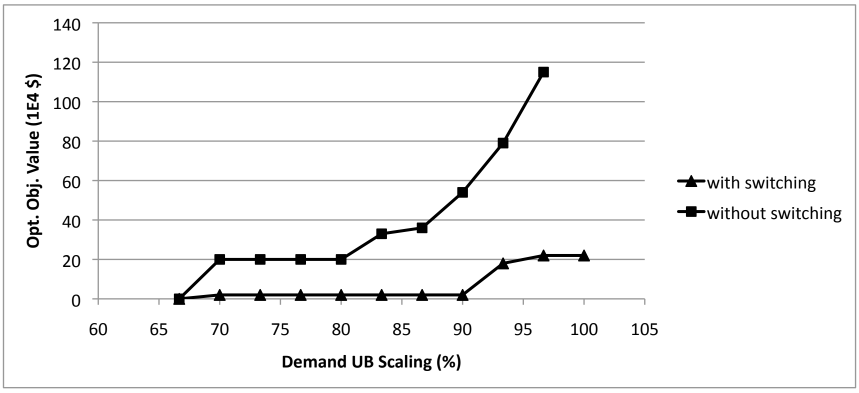

Fig. 6 Optimal Investment Cost for IEEE14 Test Case with Scaled Demand

lines. We fixed the capacity on the low capacity lines, and scaled the capacity on the high capacity lines relative to their nominal capacity. The optimal cost as a function of the scaled line capacity is shown in Figure 7.

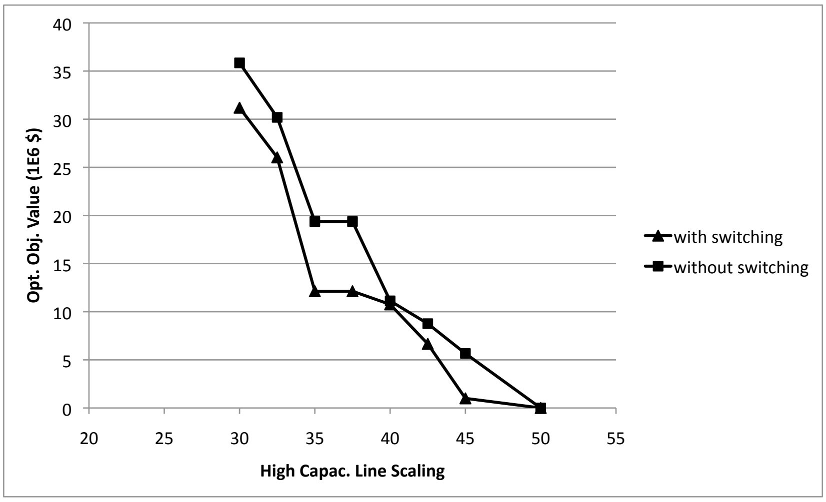

Fig. 7 Optimal Investment Cost for IEEE24 Test Case with Scaled Line Capacities

It is interesting how similar the shape of the curves in the plot in Figure 7 are to the curves in the plot in Figure 5. Essentially, scaling the upper bound on the demand and scaling the transmission capacities are two different ways of controlling the congestion level in the network. Similar levels of congestion require similar levels of investment, regardless of the source of the congestion.

6 Conclusion

A robust model for the transmission expansion problem has been presented in which there is uncertainty in both possible line failures and nodal demands, and transmission switching is used as a recovery action. The box uncertainty set that we have chosen to model demand uncertainty can represent uncertainty in loads and uncertainty in renewable generation. This robust uncertainty model is appropriate in the transmission expansion setting, as probabilities about possible failures or demand scenarios is typically not available, and avoiding blackout events is a critical priority. Within this conservative planning framework, it is shrewd to consider recovery actions such as transmission switching that would introduce flexibility, allowing the operator to achieve maintain resilient operation for a lower investment cost. The algorithm presented here could be used as a tool to evaluate the potential cost savings of allowing transmission switching as a recovery action for various ranges on the demand/renewable generation.

We have presented an algorithm that is based on the Benders' decomposition framework, but utilizes a novel oracle for identifying unsurvivable contingency-demand events. The development of the oracle allows enables the Benders' routine to be used when the number of all contingency-demand pairs is too large to practically use a naive Benders' decomposition.

In our future work, it would be interesting to explore alternate types of uncertainty sets for the demand and contingencies. For examples, contingencies of larger sizes may be explored, or polyhedral demand uncertainty sets. Additionally, we would like to explore approaches to scaling this algorithm up to larger networks. Furthermore, interesting extensions to the model presented here might include the addition of generator investment decisions, decisions regarding the order in which equipment is installed, or consideration of operational costs.

Appendix

The explicit formulation of the deterministic transmission expansion problem is defined as follows, where the contingency $\bar{c}$ and the demand vector $\bar{d}$ are known.

The following notation is defined.

Sets and indices

$\mathcal{N}$	set of buses, i.e., nodes in the network.
$\mathcal{G}$	set of all generating units. Each generator $g \in \mathcal{G}$ is located at exactly one bus $i \in \mathcal{N}$.
$\mathcal{G}_i$	set of generating units at bus $i \in \mathcal{N}$.
$i(g)$	the bus i such that $g \in \mathcal{G}_i$.
$\mathcal{E}^{\text{cand}}$	set of all candidate transmission elements
$\mathcal{E}$	set of all existing and candidate transmission elements. Power may flow in either direction on an arc, but an arbitrary direction is chosen for each arc for convenience of notation.
$\mathcal{E}_i^{\text{out}}$	set of existing and candidate transmission lines directed out of bus $i \in \mathcal{N}$.
$\mathcal{E}_i^{\text{in}}$	set of existing and candidate transmission lines directed into bus $i \in \mathcal{N}$.
$h(e)$	bus that transmission element e is directed into, i.e., the head of e.
$t(e)$	bus that transmission element e is directed out of, i.e., the tail of e.

Parameters

B_e	electrical susceptance on line $e \in \mathcal{E}$.
$\bar{c}_e$	binary parameter indicating the availability of transmission line e in the contingency. $\bar{c}_e = 1$ indicates that the transmission line e is contained in the contingency and is *not* available.
$P_g^{\max}$	upper bound on the power output at generator $g \in \mathcal{G}$.
$\bar{d}_i$	load or renewable generation at bus i. $\bar{d}_i > 0$ represents true demand, and $\bar{d}_i < 0$ represents renewable generation.
$\theta^{\min}, \theta^{\max}$	lower and upper bounds, respectively, on phase angle values.
b_e^{line}	investment cost of building transmission line $e \in \mathcal{E}^{\text{cand}}$.
b_e^{switch}	investment cost of building transmission switching equipment on line $e \in \mathcal{E}$.
F_e	capacity on power flow on transmission line $e \in \mathcal{E}$.

Variables

x_e^{line}	binary transmission expansion variable, equals 1 if transmission line e is built, for all $e \in \mathcal{E}^{\text{cand}}$.
x_e^{switch}	binary switching equipment investment variable, equals 1 if transmission switching equipment is built on line e, for all $e \in \mathcal{E}$
p_g	power output at generator g, for all $g \in \mathcal{G}$.
f_e	power flow on transmission element e, for all $e \in \mathcal{E}$.
θ_i	phase angle of bus i, for all $i \in \mathcal{N}$.
r_i	net power injection at node i, for all $i \in \mathcal{N}$.
w_e	binary transmission switching variable, equals 1 if transmission line is switched out (i.e., effectively removed), for all $e \in \mathcal{E}$.

The explicit deterministic transmission expansion problem is as follows:

$$\min \quad \sum_{e\in\mathcal{E}^{\text{cand}}} b_e^{\text{line}} x_e^{\text{line}} + \sum_{e\in\mathcal{E}} b_e^{\text{switch}} x_e^{\text{switch}} \tag{11a}$$

$$\sum_{g\in G_i} p_g + \sum_{e\in\mathcal{E}_i^{\text{in}}} f_e - \sum_{e\in\mathcal{E}_i^{\text{out}}} f_e - r_i = 0 \quad \forall i \in N \tag{11b}$$

$$\theta^{\min} \le \theta_i \le \theta^{\max} \quad \forall i \in \mathcal{N} \tag{11c}$$

$$-F_e(1-\bar{c}_e - w_e) \le f_e \le F_e(1-\bar{c}_e - w_e) \quad \forall e \in \mathcal{E} \tag{11d}$$

$$w_e \le 1 - \bar{c}_e \quad \forall e \in \mathcal{E} \tag{11e}$$

$$w_e \le x_e^{\text{switch}} \quad \forall e \in \mathcal{E} \tag{11f}$$

$$0 \le p_g \le P_g^{\max} \quad \forall g \in \mathcal{G} \tag{11g}$$

$$B_e\big(\theta_{t(e)} - \theta_{h(e)}\big) - f_e \le M(\bar{c}_e + w_e) \quad \forall e \in E\backslash\mathcal{E}^{\text{cand}} \tag{11h}$$

$$B_e\big(\theta_{t(e)} - \theta_{h(e)}\big) - f_e \ge -M(\bar{c}_e + w_e) \quad \forall e \in E\backslash\mathcal{E}^{\text{cand}} \tag{11i}$$

$$B_e\big(\theta_{t(e)} - \theta_{h(e)}\big) - f_e - M(1 - x_e^{\text{line}} + \bar{c}_e + w_e) \le 0 \quad \forall e \in \mathcal{E}^{\text{cand}} \tag{11j}$$

$$B_e\big(\theta_{t(e)} - \theta_{h(e)}\big) - f_e + M(1 - x_e^{\text{line}} + \bar{c}_e + w_e) \ge 0 \quad \forall e \in \mathcal{E}^{\text{cand}} \tag{11k}$$

$$-F_e x_e^{\text{line}} \le f_e \le F_e x_e^{\text{line}} \quad \forall e \in \mathcal{E}^{\text{cand}} \tag{11l}$$

$$r_i = \bar{d}_i \quad \forall i \in \mathcal{N} \tag{11m}$$

$$x_e^{\text{line}} \in \{0,1\} \quad \forall e \in \mathcal{E}^{\text{cand}} \tag{11n}$$

$$x_e^{\text{switch}} \in \{0,1\} \quad \forall e \in \mathcal{E} \tag{11o}$$

$$w_e \in \{0,1\} \quad \forall e \in \mathcal{E} \tag{11p}$$

The objective (11a) minimizes the total investment cost of building new transmission lines and transmission switching equipment.

Constraint (11b) requires that power flow balance must be met at each node. Constraint (11c) requires that the node phase angles are within the upper and lower bounds. Constraint (11d) requires that the line flows be within upper and lower bounds if the line is available. The power flow is forced to 0 if the power line is disrupted in the contingency, or if the transmission line is switched out. Constraint (11e) requires that a line can only be switched out if that line is not disrupted in the contingency. Constraint set (11f) requires that a line cannot be switched out unless transmission switching equipment has been installed on that line. Constraint (11g) specifies that the power output at a generator must be less than its upper bound. The lower bound on the generator dispatch is set equal to 0 because it is assumed that the generator is allowed to be operated in regimes that are inefficient but allowable for short periods when the system is stressed. Constraints (11h)-(11i) specify that, for all existing transmission lines, the DC power flow constraints must be enforced if the transmission line is not contained in the contingency and is not switched out.

Constraints (11j)-(11k) specifies that, for all candidate transmission lines, if the line is built, the DC power flow constraints must be enforced if the transmission line is not contained in the contingency and is not switched out. Constraint (11l) specifies that, for all candidate transmission lines, the power flow must be 0 for all transmission lines that are not built. Constraint (11m) specifies that net power flow injection at each node must be equal to the power demand or renewable generation at that node.

Acknowledgements This work was supported in part by an NSF Graduate Student Fellowship.

Sandia National Laboratories' Laboratory-Directed Research funded portions of this work. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

References

1. Alguacil, N., Arroyo, J.M., Carrión, M.: Transmission network expansion planning under deliberate outages. In: Handbook of Power Systems I, pp. 365–389. Springer (2010)
2. Arroyo, J., Fernández, F.: A genetic algorithm approach for the analysis of electric grid interdiction with line switching. In: Intelligent System Applications to Power Systems, 2009. ISAP'09. 15th International Conference on, pp. 1–6. IEEE (2009)
3. Binato, S., De Oliveira, G.C., De Araújo, J.L.: A greedy randomized adaptive search procedure for transmission expansion planning. IEEE Transactions on Power Systems **16**(2), 247–253 (2001)
4. Center for the New Energy Economy: State renewable portfolio standards hold steady or expand in 2013 session (2013). http://www.aeltracker.org/graphics/uploads/2013-State-By-State-RPS-Analysis.pdf
5. Choi, J., Tran, T., El-Keib, A., Thomas, R., Oh, H., Billinton, R.: A method for transmission system expansion planning considering probabilistic reliability criteria. Power Systems, IEEE Transactions on **20**(3), 1606–1615 (2005)
6. Da Silva, E.L., Ortiz, J.A., de Oliveira, G.C., Binato, S.: Transmission network expansion planning under a tabu search approach. IEEE Transactions on Power Systems **16**(1), 62–68 (2001)
7. Delgadillo, A., Arroyo, J., Alguacil, N.: Analysis of electric grid interdiction with line switching. IEEE Trans. on Power Syst. **25**(2), 633–641 (2010)
8. Fisher, E., O'Neill, R., Ferris, M.: Optimal transmission switching. IEEE Trans. on Power Syst. **23**(3), 1346–1355 (2008)
9. Freris, L.L., Sasson, A.M.: Investigation of the load-flow problem. In: Proceedings of the Institution of Electrical Engineers, vol. 115, pp. 1459–1470. IET (1968)

10. Garver, L.L.: Transmission network estimation using linear programming. IEEE Transactions on Power Apparatus and Systems **PAS-89**(7), 1688–1697 (1970)
11. Grigg, C., Wong, P., Albrecht, P., Allan, R., Bhavaraju, M., Billinton, R., Chen, Q., Fong, C., Haddad, S., Kuruganty, S., et al.: The IEEE reliability test system-1996. IEEE Trans. on Power Systems **14**(3), 1010–1020 (1999). http://www.ee.washington.edu/research/pstca/
12. Gupte, A., Ahmed, S., Cheon, M.S., Dey, S.: Solving mixed integer bilinear problems using milp formulations. SIAM Journal on Optimization **23**(2), 721–744 (2013)
13. Hedman, K., Ferris, M., O'Neill, R., Fisher, E., Oren, S.: Co-optimization of generation unit commitment and transmission switching with N-1 reliability. IEEE Transactions on Power Syst. **25**(2), 1052–1063 (2010)
14. Hedman, K., O'Neill, R., Fisher, E., Oren, S.: Optimal transmission switching with contingency analysis. IEEE Trans. on Power Syst. **24**(3), 1577–1586 (2009)
15. Hedman, K., Oren, S., O'Neill, R.: Optimal transmission switching: economic efficiency and market implications. J. of Regulatory Econ. **40**(2), 111–140 (2011)
16. Hemmati, R., Hooshmand, R.A., Khodabakhshian, A.: Market based transmission expansion and reactive power planning with consideration of wind and load uncertainties. Renewable and Sustainable Energy Reviews **29**, 1–10 (2014)
17. Jabr, R.A.: Robust transmission network expansion planning with uncertain renewable generation and loads. IEEE Transactions on Power Systems **28**(4), 4558–4567 (2013)
18. Jiang, R., Zhang, M., Li, G., Guan, Y.: Two-stage robust power grid optimization problem. submitted to Journal of Operations Research (2010)
19. Khanabadi, M., Ghasemi, H., Doostizadeh, M.: Optimal transmission switching considering voltage security and n-1 contingency analysis. IEEE Trans. on Power Syst. **28**(1) (2013)
20. Khodaei, A., Shahidehpour, M.: Transmission switching in security-constrained unit commitment. IEEE Transactions on Power Systems **25**(4), 1937–1945 (2010)
21. Khodaei, A., Shahidehpour, M., Kamalinia, S.: Transmission switching in expansion planning. IEEE Trans. on Power Syst. **25**(3), 1722–1733 (2010)
22. Latorre, G., Cruz, R.D., Areiza, J.M., Villegas, A.: Classification of publications and models on transmission expansion planning. Power Systems, IEEE Transactions on **18**(2), 938–946 (2003)
23. Li, M., Luh, P.B., Michel, L.D., Zhao, Q., Luo, X.: Corrective line switching with security constraints for the base and contingency cases. Power Systems, IEEE Transactions on **27**(1), 125–133 (2012)
24. López, J.Á., Ponnambalam, K., Quintana, V.H.: Generation and transmission expansion under risk using stochastic programming. Power Systems, IEEE Transactions on **22**(3), 1369–1378 (2007)
25. Romero, N., Xu, N., Nozick, L.K., Dobson, I., Jones, D.: Investment planning for electric power systems under terrorist threat. Power Systems, IEEE Transactions on **27**(1), 108–116 (2012)
26. Romero, R., Gallego, R., Monticelli, A.: Transmission system expansion planning by simulated annealing. IEEE Transactions on Power Systems **11**(1), 364–369 (1996)
27. Romero, R., Monticelli, A., Garcia, A., Haffner, S.: Test systems and mathematical models for transmission network expansion planning. IEE Proceedings-Generation, Transmission and Distribution **149**(1), 27–36 (2002)
28. Shirokikh, O., Sorokin, A., Boginski, V.: A note on transmission switching in electric grids with uncertain line failures. Energy Systems **4**(4), 419–430 (2013)
29. Silva, I.J., Rider, M.J., Romero, R., Murari, C.A.: Transmission network expansion planning considering uncertainty in demand. Power Systems, IEEE Transactions on **21**(4), 1565–1573 (2006)
30. Sorokin, A., Portela, J., Pardalos, P.M.: Algorithms and models for transmission expansion planning. In: Handbook of Networks in Power Systems I, pp. 395–433. Springer (2012)
31. Villumsen, J.C., Bronmo, G., Philpott, A.B.: Line capacity expansion and transmission switching in power systems with large-scale wind power. IEEE Trans. on Power Syst. **28**(2), 731–739 (2013)

32. Villumsen, J.C., Philpott, A.B.: Investment in electricity networks with transmission switching. Europ. J. of Operational Res. **222**(2), 377–385 (2012)
33. Wu, P., Cheng, H., Xing, J.: The interval minimum load cutting problem in the process of transmission network expansion planning considering uncertainty in demand. Power Systems, IEEE Transactions on **23**(3), 1497–1506 (2008)
34. Xu, Z., Dong, Z., Wong, K.: Transmission planning in a deregulated environment. IEE Proceedings-Generation, Transmission and Distribution **153**(3), 326–334 (2006)
35. Yu, H., Chung, C., Wong, K.: Robust transmission network expansion planning method with taguchi's orthogonal array testing. Power Systems, IEEE Transactions on **26**(3), 1573–1580 (2011)
36. Yu, H., Chung, C., Wong, K., Zhang, J.: A chance constrained transmission network expansion planning method with consideration of load and wind farm uncertainties. Power Systems, IEEE Transactions on **24**(3), 1568–1576 (2009)
37. Zhao, L., Zeng, B.: An exact algorithm for power grid interdiction problem with line switching. Submitted, available in optimization-online, University of South Florida (2011)
38. Zhao, L., Zeng, B.: Robust unit commitment problem with demand response and wind energy. In: Power and Energy Society General Meeting, 2012 IEEE, pp. 1–8. IEEE (2012)